Duration: 55 minutes

1.a) (25%) Use the method of separation of variables to solve the 1-d heat IBVP:

$$
\begin{array}{ll}
\text{PDE: } u_t = \alpha^2 u_{xx} & 0 < x < 1,\ 0 < t < \infty \\
\text{BCs: } u(0,t) = 0 & 0 < t < \infty \\
\qquad\ u(1,t) = 0 & 0 < t < \infty \\
\text{IC: } u(x,0) = \phi(x) & 0 \le x \le 1
\end{array}
$$

(α is a positive constant). **Show all details.**

b)(10%) Deduce the solution of the nonhomogeneous problem:

$$
\begin{array}{ll}
\text{PDE: } u_t = u_{xx} - u + x & 0 < x < 1,\ 0 < t < \infty \\
\text{BCs: } u(0,t) = 0 & 0 < t < \infty \\
\qquad\ u(1,t) = 1 & 0 < t < \infty \\
\text{IC: } u(x,0) = \psi(x) & 0 \le x \le 1
\end{array}
$$

__*Hint:*__ $u(x,t) = x + e^{-t} w(x,t)$.

2.(30%) Use the Fourier Transform to solve the IVP

$$
\begin{array}{ll}
\text{PDE: } u_t = \alpha^2 u_{xx} - \beta u & -\infty < x < \infty,\ 0 < t < \infty \\
\text{IC: } u(x,0) = \phi(x) & -\infty < x < \infty
\end{array}
$$

(α, β are positive constants.)

3.(35%) Solve by means of the Laplace transform the IBVP:

$$
\begin{array}{ll}
\text{PDE: } u_t + x u_x = x & 0 < x < \infty,\ 0 < t < \infty \\
\text{BC: } u(0,t) = 0 & 0 < t < \infty \\
\text{IC: } u(x,0) = 0 & 0 < x < \infty
\end{array}
$$

e^{xs^2} x

$x^2 s$ $2x e^{xs}$

$=$ $-\beta t$